

An Approach to Fuzzy Modeling of Anti-lock Braking Systems

Radu-Codruț David¹, Ramona-Bianca Grad¹, Radu-Emil Precup¹,
Mircea-Bogdan Rădac¹, Claudia-Adina Dragoș¹, and Emil M. Petriu²

¹ Department of Automation and Applied Informatics, “Politehnica” University of Timisoara,
Bd. V. Parvan 2, 300223 Timisoara, Romania

davidradu@gmail.com, bibi23grad@yahoo.com, radu.precup@aut.upt.ro,
mircea.radac@aut.upt.ro, claudia.dragos@aut.upt.ro

² School of Electrical Engineering and Computer Science, University of Ottawa,
800 King Edward, Ottawa, ON, K1N 6N5 Canada
petriu@eecs.uottawa.ca

Abstract. This paper proposes an approach to fuzzy modeling of Anti-lock Braking Systems (ABSs). The local state-space models are derived by the linearization of the nonlinear ABS process model at ten operating points. The Takagi-Sugeno (T-S) fuzzy models are obtained by the modal equivalence principle, where the local state-space models are the rule consequents. The optimization problems are defined in order to minimize the objective functions expressed as the squared modeling errors, and the variables of these functions are a part of the parameters of input membership functions. Simulated Annealing algorithms are implemented to solve the optimization problems and to obtain optimal T-S fuzzy models. Real-time experimental results are included to validate the new optimal T-S fuzzy models for an ABS laboratory equipment.

Keywords: Anti-lock Braking Systems, optimization, real-time experiments, simulated annealing, Takagi-Sugeno fuzzy models.

1 Introduction

The development of fuzzy models for Anti-lock Braking Systems (ABSs) is a challenging problem because of the importance of these safety subsystems in modern cars and of their nonlinearities. Several approaches to fuzzy modeling of ABSs are given in the literature with the aim of slip control. An approach to ABS process identification and robust adaptive control connected to an active suspension system by hierarchical T-S fuzzy-neural models is discussed in [1]. A T-S fuzzy model of deceleration based on analyzing the braking process and dynamic model of vehicle and wheel is proposed in [2]. A quarter vehicle braking model with four-degrees of freedom subject to irregular excitation from a road surface is offered in [3] and applied to ABS fuzzy control. Two discrete-time dynamic Takagi-Sugeno (T-S) fuzzy models of ABS processes are suggested in [4] using the computation of the minimum and maximum values of input variables and the local linearization at several operating points. Com-

binations of fuzzy models applied to intelligent ABS controllers based on fuzzy control, neural networks and sliding mode control are given in [5–8].

This paper offers a simple approach to fuzzy modeling of ABSs. Our approach starts with the derivation of an initial discrete-time T-S fuzzy model of the process by the modal equivalence principle; this fuzzy model is characterized by a set of local linearized state-space models of the process which are placed in the rule consequents. A part of the parameters of input membership functions (m.f.s) is optimized by a Simulated Annealing (SA) algorithm in order to solve the optimization problems which aim the minimization of the sum of squared modeling errors expressed as difference between the process output (the wheel slip) and the T-S fuzzy model output. The reason to select the SA algorithm is, as shown in [9] for the optimal tuning of fuzzy controllers, its ability to deal with non-convex objective functions (o.f.s).

Our approach is different to the approaches presented in the literature because it starts with the first-principle mathematical model of the process and it offers a strong advantage by the verification of the performance of the optimal T-S fuzzy model in terms of real-time experiments on the ABS laboratory equipment. Although our approach cannot guarantee that the global minimum of o.f. is reached, this paper shows that a serious decrease of o.f. is exhibited, clearly indicating the performance improvement offered by our T-S fuzzy model.

The paper treats the following topics. Section 2 is dedicated to the mathematical modeling of the process and to the design of discrete-time dynamic T-S fuzzy models with focus on an ABS laboratory equipment. The main aspects concerning the implementation of our SA algorithm are discussed in Section 3. Real-time experimental results are given in Section 4 to validate the new optimal T-S fuzzy models. The conclusions are highlighted in Section 5.

2 ABS Process Models

The nonlinear state-space model of the ABS laboratory equipment [10] is derived starting with the first-principle equations [10, 11]

$$\begin{aligned}
 J_1 \dot{x}_1 &= F_n r_1 \mu(\lambda) - d_1 x_1 - M_{10} - M_1, \\
 J_2 \dot{x}_2 &= -F_n r_2 \mu(\lambda) - d_2 x_2 - M_{20},
 \end{aligned}
 \tag{1}$$

where λ is the longitudinal slip (the wheel slip), J_1 and J_2 are the inertia moments of wheels, x_1 and x_2 are the angular velocities, d_1 and d_2 are the friction coefficients in wheels' axes, M_{10} and M_{20} are the static friction torques that oppose the normal rotation, M_1 is the brake torque, r_1 and r_2 are the radii of wheels, F_n is the normal force that the upper wheel pushes upon the lower wheel, $\mu(\lambda)$ is the friction coefficient, and \dot{x}_1 and \dot{x}_2 are the angular accelerations of wheels. The identification by measurements and experiments leads to the parameter values [11]:

$$\begin{aligned}
 r_1 = r_2 = 0.99 \text{ m}, F_n = 58.214 \text{ N}, J_1 = 7.53 \cdot 10^{-3} \text{ kg m}^2, J_2 = 25.6 \cdot 10^{-3} \text{ kg m}^2, \\
 d_1 = 1.1874 \cdot 10^{-4} \text{ kg m}^2/\text{s}, d_2 = 2.1468 \cdot 10^{-4} \text{ kg m}^2/\text{s}, M_{10} = 0.0032 \text{ N m}, \\
 M_{20} = 0.0925 \text{ N m}.
 \end{aligned} \tag{2}$$

The wheel slip and the nonlinear factor $S(\lambda)$ are expressed as

$$\lambda = (r_2 x_2 - r_1 x_1) / (r_2 x_2), x_2 \neq 0, S(\lambda) = \mu(\lambda) / \{L[\sin \varphi - \mu(\lambda) \cos \varphi]\}, \tag{3}$$

where $L = 0.37 \text{ m}$ is arm's length which fixes the upper wheel, and $\varphi = 65.61^\circ$ is the angle between the normal direction in wheels' contact point and L 's direction.

The nonlinear state-space equations of the process are

$$\begin{aligned}
 \dot{x}_1 &= S(\lambda)(c_{11}x_1 + c_{12}) + c_{13}x_1 + c_{14} + (c_{15}S(\lambda) + c_{16})s_1M_1, \\
 \dot{x}_2 &= S(\lambda)(c_{21}x_1 + c_{22}) + c_{23}x_2 + c_{24} + c_{25}S(\lambda)s_1M_1, \\
 \dot{M}_1 &= c_{31}(b(u) - M_1),
 \end{aligned} \tag{4}$$

where u is the control signal applied to the DC motor which drives the upper wheel and the nonlinear model of the actuator is highlighted in the third equation. The expressions of the parameters in (4) are [11]

$$\begin{aligned}
 c_{11} = r_1 d_1 / J_1, c_{12} = (M_{10} + M_g)r_1 / J_1, c_{13} = -d_1 / J_1, c_{14} = -M_{10} / J_1, \\
 c_{15} = r_1 / J_1, c_{16} = -1 / J_1, c_{21} = -r_2 d_1 / J_2, c_{22} = -(M_{10} + M_g)r_2 / J_2, \\
 c_{23} = -d_2 / J_2, c_{24} = -M_{20} / J_2, c_{25} = -r_2 / J_2.
 \end{aligned} \tag{5}$$

The introduction of λ as controlled output in the model (4) is done by the substitution of x_1 from (3). The state-space equations of ABS process are

$$\begin{aligned}
 \dot{\lambda} &= Z_1(\lambda, x_2)\lambda + Z_3(\lambda, x_2)M_1 + Z_{20}(\lambda, x_2), \\
 \dot{x}_2 &= Z_{40}(\lambda)x_2 + Z_5(\lambda)M_1 + Z_6(\lambda), \\
 \dot{M}_1 &= c_{31}(b(u) - M_1),
 \end{aligned} \tag{6}$$

they point out the state vector \mathbf{x} :

$$\mathbf{x} = [\lambda \quad x_2 \quad M_1]^T, \tag{7}$$

where T indicates the matrix transposition, and the variables are defined in [4, 11].

The steps of our modeling approach are:

- the definition of m.f.s of the input variables λ , x_2 and M_1 ,
- the derivation of an initial T-S fuzzy model of the process, with the state variables λ , x_2 and M_1 as input variables, and the discrete-time state-space process models with the matrices $\mathbf{A}_{d,i}$, $\mathbf{B}_{d,i}$, $\mathbf{C}_{d,i}$ and $\mathbf{D}_{d,i}$, $i = 1 \dots 10$, in the rule consequents,
- the definition of the optimization problem where the vector variable of the o.f. consists of a part of the parameters of input m.f.s,

- the application of SA algorithm to obtain the optimal input m.f. parameters which lead to the optimal T-S fuzzy model.

The derivation of the initial T-S fuzzy model starts with the setting of the largest domains of variation of the three state variables in all ABS operating regimes [4, 11]:

$$0.1 \leq \lambda \leq 1, 20 \leq x_2 \leq 178, 0 \leq M_1 \leq 10. \quad (8)$$

The fuzzification in the T-S fuzzy is done using the linguistic terms assigned to the input variables and defined as follows. The first input variable, λ , uses five linguistic terms, $LT_{\lambda,j}, j=1..5$, with the triangular m.f.s

$$\begin{aligned}
 &\mu_{LT_{\lambda,1}} : [0,0.2] \rightarrow [0,1], \mu_{LT_{\lambda,2}} : [0.1,0.4] \rightarrow [0,1], \mu_{LT_{\lambda,3}} : [0.2,0.8] \rightarrow [0,1], \\
 &\mu_{LT_{\lambda,4}} : [0.4,1] \rightarrow [0,1], \mu_{LT_{\lambda,5}} : [0.8,1.1] \rightarrow [0,1], \\
 &\mu_{LT_{\lambda,j}}(x) = \begin{cases} 0, & x < a_{\lambda,j}, \\ 1 + (x - b_{\lambda,j}) / (b_{\lambda,j} - a_{\lambda,j}), & x \in a_{\lambda,j} \leq x < b_{\lambda,j}, \\ 1 - (x - b_{\lambda,j}) / (c_{\lambda,j} - b_{\lambda,j}), & x \in b_{\lambda,j} \leq x < c_{\lambda,j}, \\ 0, & x \geq c_{\lambda,j}, \end{cases} \quad (9) \\
 &a_{\lambda,j} < b_{\lambda,j} < c_{\lambda,j}, j = 1..5.
 \end{aligned}$$

The parameters $a_{\lambda,j}, j=1..5$, and $c_{\lambda,j}, j=1..5$, are variable and they belong to the vector variable \mathbf{p} of o.f., and the parameters $b_{\lambda,j}, j=1..5$, which stand for the modal values of m.f.s, are fixed: $b_{\lambda,1} = 0.1, b_{\lambda,2} = 0.2, b_{\lambda,3} = 0.4, b_{\lambda,4} = 0.8$ and $b_{\lambda,5} = 1$. The second input variable, x_2 , uses two linguistic terms, $LT_{x_2,j}, j=1..2$, with the triangular m.f.s of type (9), $\mu_{LT_{x_2,1}} : [0,150] \rightarrow [0,1]$ and $\mu_{LT_{x_2,2}} : [50,180] \rightarrow [0,1]$. The parameters $a_{x_2,j}, j=1..2$, and $c_{x_2,j}, j=1..2$, are variable and they belong to \mathbf{p} , and the parameters $b_{x_2,j}, j=1..2$, are fixed: $b_{x_2,1} = 50$ and $b_{x_2,2} = 150$. The third input variable, M_1 , uses one linguistic term, $LT_{M_1,1}$, with the triangular m.f. of type (9), $\mu_{LT_{M_1,1}} : [0,11] \rightarrow [0,1]$. The parameters $a_{M_1,1}$ and $c_{M_1,1}$ are and they belong to \mathbf{p} , and the parameter $b_{M_1,1}$ is fixed: $b_{M_1,1} = 10$. Other m.f. shapes can be used in different industrial applications [12–17].

The complete rule base of the discrete-time dynamic T-S fuzzy model of the process consists of the rules $R^i, i = 1..10$, expressed as:

$$\begin{aligned}
 R^1 : & \text{IF } \lambda \text{ IS } LT_{\lambda,1} \text{ AND } x_2 \text{ IS } LT_{x_2,1} \text{ AND } M_1 \text{ IS } LT_{M_1,1} \text{ THEN } \begin{cases} \mathbf{x}_{k+1} = \mathbf{A}_{d,1} \mathbf{x}_k + \mathbf{B}_{d,1} u_k, \\ y_{k,m} = \mathbf{C}_{d,1} \mathbf{x}_k, \end{cases} \quad (10) \\
 & \dots \\
 R^{10} : & \text{IF } \lambda \text{ IS } LT_{\lambda,5} \text{ AND } x_2 \text{ IS } LT_{x_2,2} \text{ AND } M_1 \text{ IS } LT_{M_1,1} \text{ THEN } \begin{cases} \mathbf{x}_{k+1} = \mathbf{A}_{d,10} \mathbf{x}_k + \mathbf{B}_{d,10} u_k, \\ y_{k,m} = \mathbf{C}_{d,10} \mathbf{x}_k, \end{cases}
 \end{aligned}$$

where the discrete-time state-space models in the rule consequents are obtained by the discretization of the continuous-time state-space linearized models at ten operating points. The state-space model matrices in the rule consequents of R^1 and R^{10} are

$$\begin{aligned}
 \mathbf{A}_{d,1} &= \begin{bmatrix} 0.9441 & -0.0025 & 0.0194 \\ -1.0335 & 1.0012 & -0.0601 \\ 0 & 0 & 0.8157 \end{bmatrix}, \mathbf{B}_{d,1} = \begin{bmatrix} 0.0021 \\ -0.0059 \\ 0.1843 \end{bmatrix}, \mathbf{C}_{d,1} = [1 \ 0 \ 0], \\
 \mathbf{A}_{d,10} &= \begin{bmatrix} 1.0041 & -0.0003 & 0.0069 \\ -0.3192 & 1 & -0.0519 \\ 0 & 0 & 0.8157 \end{bmatrix}, \mathbf{B}_{d,10} = \begin{bmatrix} 0.0007 \\ -0.0054 \\ 0.1843 \end{bmatrix}, \mathbf{C}_{d,10} = [1 \ 0 \ 0].
 \end{aligned} \tag{11}$$

The modal equivalence principle is applied to obtain the initial fuzzy model (10) with the rule consequent parameters given in (11) as the coordinates of the operating points are represented by the modal values of input m.f.s. The sampling period $T_s = 0.01$ s is used in this context, and k in (10) indicates the number of current sampling interval. The SUM and PROD operators are used in the inference engine, and the weighted average method is employed for defuzzification. Different operators and defuzzification lead to modified nonlinear input-output model maps [18–21].

3 Simulated Annealing Algorithm

SA is applied to solve the optimization problem

$$\mathbf{p}^* = \arg \min_{\mathbf{p} \in D} J(\mathbf{p}), \tag{12}$$

where the parameter vector of the fuzzy model (the vector variable of o.f.) is

$$\mathbf{p} = [a_{\lambda,1} \ c_{\lambda,1} \ a_{\lambda,2} \ c_{\lambda,2} \ a_{\lambda,3} \ c_{\lambda,3} \ a_{\lambda,4} \ c_{\lambda,4} \ a_{\lambda,5} \ c_{\lambda,5} \ a_{x_2,1} \ c_{x_2,1} \ a_{x_2,2} \ c_{x_2,2} \ a_{M_1,1} \ c_{M_1,1}]^T, \tag{13}$$

$J(\mathbf{p})$ is the o.f. defined as

$$J(\mathbf{p}) = \frac{1}{N} \sum_{k=1}^N (y_k(\mathbf{p}) - y_{k,m}(\mathbf{p}))^2 = \frac{1}{N} \sum_{k=1}^N (e_{k,m}(\mathbf{p}))^2, \tag{14}$$

\mathbf{p}^* is the optimal parameter vector of the fuzzy model and the solution to the optimization problem (12), $y_k(\mathbf{p}) = \lambda_k(\mathbf{p})$ is the process output at k^{th} sampling interval, $y_{k,m}(\mathbf{p})$ is the fuzzy model output, $e_{k,m}(\mathbf{p}) = y_k(\mathbf{p}) - y_{k,m}(\mathbf{p})$ is the modeling error, N is the length of the time horizon, and D is the feasible domain of (12).

The SA algorithm is adapted from the general SA algorithms proposed in [22, 23] and from the SA suggested in [9] and applied to the optimal tuning of fuzzy controller parameters. The steps of our SA algorithm are

- *Step 1.* Set $\mu = 0$, where μ is the iteration number, $s_{r_{\max}}$ and $r_{r_{\max}}$ (the maximum success and rejection rates defined in [24]) with the initial values $s_r = 0$ and $r_r = 0$, and the minimum temperature θ_{\min} . Choose the initial temperature θ_0 .
- *Step 2.* Generate a random initial solution φ and compute its fitness value $C(\varphi)$, where C is the fitness function.
- *Step 3.* Generate a probable solution ψ by disturbing φ , and evaluate the fitness value $C(\psi)$.
- *Step 4.* Compute the difference

$$\Delta C_{\varphi\psi} = C(\varphi) - C(\psi). \quad (15)$$

If $\Delta C_{\varphi\psi} \leq 0$, then accept ψ as the new solution. Otherwise, set the random parameter q_n , $0 \leq q_n \leq 1$, and compute the probability of ψ to be the next solution:

$$p_\psi = \begin{cases} 1 & \text{if } \Delta C_{\varphi\psi} > 0, \\ \exp(\Delta C_{\varphi\psi} / \theta) & \text{otherwise.} \end{cases} \quad (16)$$

If $p_\psi > q_n$, then ψ is the new solution.

- *Step 5.* If the new solution is accepted, then update the new solution and C , increment s_r and set $r_r = 0$, where r_r is the rejection rate. Otherwise, increment r_r . If r_r has reached its maximum value, $r_{r_{\max}}$, the algorithm is stopped; otherwise, continue with step 6.
- *Step 6.* Increment s_r . If s_r has reached its maximum value $s_{r_{\max}}$, go to step 7; otherwise increment μ . If μ has reached its maximum value μ_{\max} , go to step 7; otherwise, go to step 2.
- *Step 7.* Alleviate the temperature according to the temperature decrement rule

$$\theta_{\mu+1} = \alpha_{cs} \theta_\mu, \alpha_{cs} = \text{const}, \alpha_{cs} < 1, \alpha_{cs} \approx 1. \quad (17)$$

- *Step 8.* If $\theta_\mu > \theta_{\min}$ then go to step 3, otherwise the algorithm is stopped indicating that it has reached the solution ψ .

This SA algorithm is mapped onto (12) by means of the following relations between the fitness and objective functions and between the parameter vectors:

$$J(\mathbf{p}) = C(\psi), J(\mathbf{p}) = C(\varphi), \mathbf{p} = \psi, \mathbf{p} = \varphi. \quad (18)$$

4 Experimental Results

A part of the results and implementation details of our modeling approach are presented as follows. The values of $r_{r_{\max}}$ and of $s_{r_{\max}}$ were set to $r_{r_{\max}} = 100$ and

$s_{r\max} = 50$. The initial temperature was chosen as $\theta_0 = 1$. The SA algorithm has stopped after 84 iterations, when the temperature value was $\theta_{84} = 0.090235$. The initial solution is [11]:

$$\mathbf{p} = [0 \quad 0.2 \quad 0.1 \quad 0.4 \quad 0.2 \quad 0.8 \quad 0.4 \quad 1 \quad 0.8 \quad 1.1 \quad 0 \quad 150 \quad 50 \quad 180 \quad 0 \quad 11]^T, \quad (19)$$

and the final solution is:

$$\mathbf{p}^* = \boldsymbol{\psi} = [0.008615 \quad 0.1614 \quad 0.09901 \quad 0.4067 \quad 0.2295 \quad 0.8471 \quad 0.4019 \quad 0.9578 \quad 0.847 \quad 1.12 \quad 0.02202 \quad 169 \quad 59.85 \quad 180.5 \quad -0.3227 \quad 11.04]^T. \quad (20)$$

The control signal u has been generated in order to cover different ranges of magnitudes. The evolution of the control signal versus time is presented in Fig. 1. Fig. 1 gives a total set of 35000 input-output data points which are separated in the training data and the validation data set for cross-validation and to assess the performance of the T-S fuzzy models. The first $N = 10000$ data points (corresponding to the time frame from 0 s to 100 s) which result from Fig. 1 are the training data set. The rest of $N = 25000$ data points (corresponding to the time frame from 100 s to 350 s) which result from Fig. 1 are the validation data set.

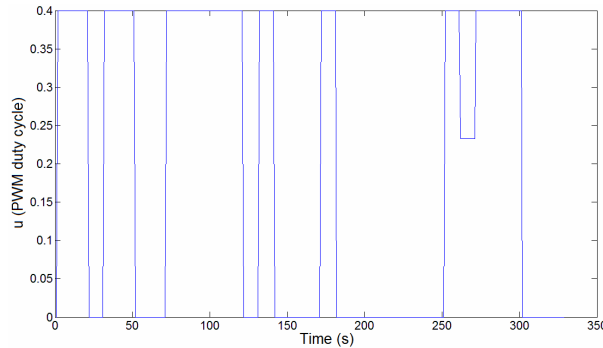


Fig. 1. Control signal u versus time, applied to real-world process and T-S fuzzy model.

The experimental results are presented in Fig. 2 and in Fig. 3 as the outputs of the real-world process (the ABS laboratory equipment), of the T-S fuzzy model before the application of the SA algorithm and of the T-S fuzzy model after the application of the SA algorithm. Fig. 2 and Fig. 3 clearly show that the performance of the T-S fuzzy model is strongly improved by the application of our SA algorithm from the point of view the modeling errors. The modeling errors are seriously alleviated in both training and validation data sets.

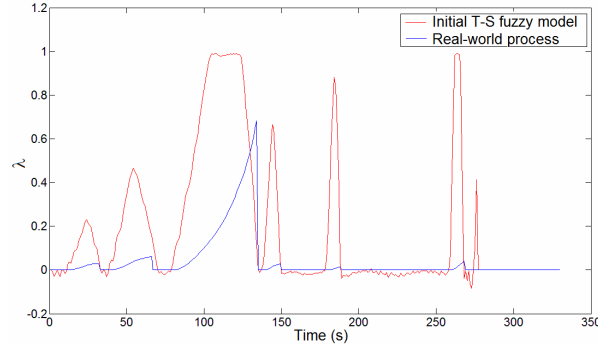


Fig. 2. Real-time experimental results: wheel slip λ versus time for initial T-S fuzzy model and for real-world process.

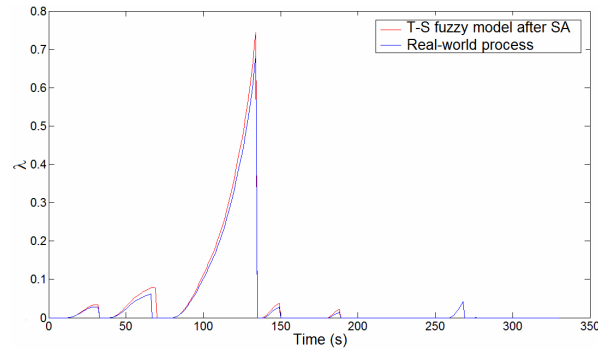


Fig. 3. Real-time experimental results: wheel slip λ versus time for T-S fuzzy model after optimization by SA algorithm and for real-world process.

Fig. 4 shows that the solution to the optimization problem (12) obtained by our SA algorithm ensures a strong decrease of the o.f. The results presented in Fig. 4 correspond to the testing data set. Although the minimum of o.f. cannot be guaranteed, Fig. 4 points out that the improvement can continue by a larger number of iterations.

5 Conclusions

This paper has proposed an approach to obtain discrete-time T-S fuzzy models dedicated to ABSs. The models result by the application of an SA algorithm which ensures the optimal computation of the parameters of T-S fuzzy models initially obtained by the modal equivalence principle.

Our approach is important because it is relatively simple, it produces models which can be implemented easily, and it can be generalized to a wide category of industrial applications. The limitation of our approach is represented by the random generation of the initial solution in the SA algorithm. However, the results are presented for the

best five runs of the SA algorithm, and they convincingly show the performance improvement of the T-S fuzzy model obtained for an ABS laboratory equipment.

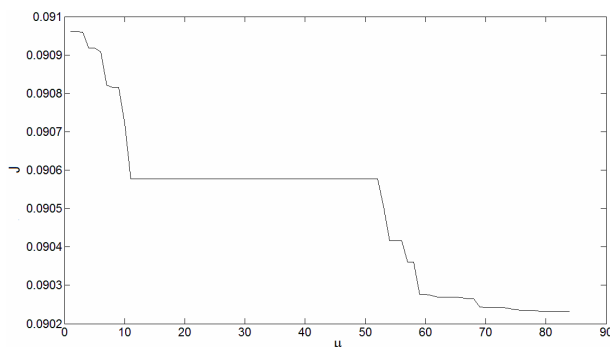


Fig. 4. Objective function J versus iteration number μ for validation data set.

Future research will deal with the extension of the SA algorithm to the modeling of multi input-multi output nonlinear systems. The inclusion of o.f. gradients in other evolutionary-based algorithms [12–17, 24], the data-driven computation of gradients and the correlation analysis will be considered.

Acknowledgments This work was supported by a grant in the framework of the Partnerships in priority areas - PN II program of the Romanian National Authority for Scientific Research ANCS, CNDI - UEFISCDI, project number PN-II-PT-PCCA-2011-3.2-0732.

References

1. Zhao, Z., Yu, Z., Sun, Z.: Research on Fuzzy Road Surface Identification and Logic Control for Anti-lock Braking System. In: Proceedings of IEEE International Conference on Vehicular Electronics and Safety, Shanghai, China, pp. 380–387 (2006)
2. Zheng, T., Ma, F., Zhang, K.: Estimation of Reference Vehicle Speed Based on T-S Fuzzy Model. In: Proceedings of International Conference on Advanced in Control Engineering and Information Science, Dali, China, vol. 15, pp. 188–193 (2011)
3. Wang, W.-Y., Chen, M.-C., Su, S.-F.: Hierarchical T-S Fuzzy-Neural Control of Anti-lock Braking System and Active Suspension in a Vehicle. *Automatica* 48, 1698–1706 (2012)
4. Precup, R. E., Spătaru, S.V., Rădac, M.-B., Petriu, E.M., Preitl, S., Dragoș, C.-A., David, R.-C.: Experimental Results of Model-Based Fuzzy Control Solutions for a Laboratory Antilock Braking System. In: Hippe, Z.S., Kulikowski, J.L., Mroczek, T., eds., *Human-Computer Systems Interaction: Backgrounds and Applications 2, Part 2*, Springer-Verlag, AICS, vol. 99, pp. 223–234 (2012)
5. Naderi, P., Farhadi, A., Mirsalim, M., Mohammadi, T.: Anti-lock and Anti-slip Braking System, Using Fuzzy Logic and Sliding Mode Controllers. In: Proceedings of IEEE Vehicle Power and Propulsion Conference, Lille, France, 6 pp. (2010)

6. Topalov, A.V., Oniz, Y., Kayacan, E., Kaynak, O.: Neuro-Fuzzy Control of Antilock Braking System Using Sliding Mode Incremental Learning Algorithm. *Neurocomputing* 74, 1883–1893 (2011)
7. Bhandari, R., Patil, S., Singh R.K.: Surface Prediction and Control Algorithms for Anti-lock Brake System. *Transportation Research Part C: Emerging Technologies* 21, 181–195 (2012)
8. Aleksendrić, D., Jakovljević, Ž., Ćirović, V.: Intelligent Control of Braking Process. *Expert Systems with Applications* 39, 11758–11765 (2012)
9. Precup, R.-E., David, R.-C., Petriu, E.M., Preitl, S., Rădac, M.-B.: Fuzzy Control Systems with Reduced Parametric Sensitivity Based on Simulated Annealing. *IEEE Transactions on Industrial Electronics* 59, 3049–3061 (2012)
10. Inteco: ABS: The Laboratory Anti-lock Braking System Controlled from PC – User Manual, Inteco Sp. z o. o., Krakow, Poland (2007)
11. Grad, R.-B.: Biologically Inspired Optimization Algorithm for Fuzzy Modeling of an Anti-lock Braking System Laboratory Equipment. M.Sc. thesis, “Politehnica” University of Timisoara, Timisoara, Romania (2012)
12. Köppen, M.: Light-Weight Evolutionary Computation for Complex Image-Processing Applications. In: *Proceedings of 6th International Conference on Hybrid Intelligent Systems*, Auckland, New Zealand, pp. 3–3 (2006)
13. Deb, K., Gupta, S., Daum, D., Branke, J., Mall, A.K., Padmanabhan, D.: Reliability-Based Optimization Using Evolutionary Algorithms. *IEEE Transactions on Evolutionary Computation* 13, 1054–1074 (2009)
14. Blažič, S., Matko, D., Škrjanc, I.: Adaptive Law with a New Leakage Term. *IET Control Theory & Applications* 4, 1533–1542 (2010)
15. Carrano, E.G., Takahashi, R.H.C., Fonseca, C.M., Neto, O.M.: Non-Linear Network Optimization – An Embedding Vector Space Approach. *IEEE Transactions on Evolutionary Computation* 14, 206–226 (2010)
16. Ko, M., Tiwari, A., Mehnen, J.: A Review of Soft Computing Applications in Supply Chain Management. *Applied Soft Computing* 10, 661–674 (2010)
17. Kudelka, M., Horak, Z., Snásel, V., Krömer, P., Platos, J., Abraham, A.: Social and Swarm Aspects of Co-authorship Network. *Logic Journal of the IGPL* 20, 634–643 (2012)
18. Johanyák, Z.C.: Student Evaluation Based on Fuzzy Rule Interpolation. *International Journal of Artificial Intelligence* 5, 37–55 (2010)
19. Vaščák, J., Madarász, L.: Adaptation of Fuzzy Cognitive Maps – A Comparison Study. *Acta Polytechnica Hungarica* 7, 109–122 (2010)
20. Castillo, O., Melin, P., Garza, A.A., Montiel, O., Sepúlveda, R.: Optimization of Interval Type-2 Fuzzy Logic Controllers Using Evolutionary Algorithms. *Soft Computing* 15, 1145–1160 (2011)
21. Schaefer, G., Hu, Q., Zhou, H., Peters, J.F., Hassanien, A.E.: Rough C-means and Fuzzy Rough C-means for Colour Quantisation. *Fundamenta Informaticae* 119, 113–120 (2012)
22. Kirkpatrick, S., Gelatt, Jr., C.D., Vecchi, M.P.: Optimization by Simulated Annealing. *Science* 20, 671–680 (1983)
23. Geman, S., Geman, D.: Stochastic Relaxation, Gibbs Distribution and the Bayesian Restoration in Images. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 6, 721–741 (1984)
24. Precup, R.-E., David, R.-C., Petriu, E.M., Rădac, M.-B., Preitl, S., Fodor, J.: Evolutionary Optimization-based Tuning of Low-cost Fuzzy Controllers for Servo Systems. *Knowledge-Based Systems* DOI: 10.1016/j.knosys.2011.07.00 (2011)